Package ‘spantest’

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Title Mean-Variance Spanning Tests

Version 1.1-3

Description Provides a comprehensive suite of portfolio spanning tests for asset pricing, such as Huberman and Kandel (1987) <doi:10.1111/j.1540-6261.1987.tb03917.x>, Gibbons et al. (1989) <doi:10.2307/1913625>, Kempf and Memmel (2006) <doi:10.1007/BF03396737>, Pesaran and Yamagata (2024) <doi:10.1093/jjfinec/nbad002>, and Gungor and Luger (2016) <doi:10.1080/07350015.2015.1019510>.

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RoxygenNote 7.3.2

Encoding UTF-8

Depends R (>= 4.1.0)

URL `https://github.com/ArdiaD/spantest`

BugReports `https://github.com/ArdiaD/spantest/issues`

Suggests rmarkdown, testthat (>= 3.0.0)

Config/testthat/edition 3

Imports Rdpack, stats, utils

RdMacros Rdpack

NeedsCompilation no

Author David Ardia [aut, cre] (ORCID: <`https://orcid.org/0000-0003-2823-782X`>),
Benjamin Seguin [aut],
Rosnel Sessinou [ctb],
Richard Luger [ctb]

Maintainer David Ardia <`david.ardia.ch@gmail.com`>

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Contents

`span_bj` *Britten–Jones Tangency-Portfolio Spanning Test (1999)*

Description

Tests whether the tangency (maximum Sharpe) portfolio of the augmented universe (benchmarks + test assets) is spanned by the benchmark assets alone. Following Britten–Jones (1999), the statistic arises from a regression of a constant on return differences and yields an F test of the tangency-spanning restriction.

Usage

```
span_bj(R1, R2)
```

Arguments

R1	Numeric matrix of benchmark returns, dimension $T \times K$.
R2	Numeric matrix of test-asset returns, dimension $T \times N$.

Details

With `X` formed from pairwise differences, the reference distribution is $F_{N,\ T-ncol(X)}$; here `ncol(X)` = $K + N - 1$. Finite-sample feasibility requires `T - (K + N - 1) >= 1`.

Value

A named list with components:

`pval` P-value for the F-statistic under the null.

`stat` Britten–Jones F-statistic.

`H0` Null hypothesis description, `"tangency portfolio spanned by benchmark"`.

References

Britten-Jones M (1999). "The Sampling Error in Estimates of Mean-Variance Efficient Portfolio Weights." *The Journal of Finance*, **54**(2), 655–671.

See Also

Other Alpha Spanning Tests: span_f1(), span_gl_a(), span_grs(), span_py()

Examples

```
set.seed(321)
R1 <- matrix(rnorm(300), 100, 3)  # benchmarks: T=100, K=3
R2 <- matrix(rnorm(200), 100, 2)  # tests:      T=100, N=2
out <- span_bj(R1, R2)
out$stat; out$pval; out$H0
```

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span_f1 *F1 Alpha-Spanning Test (Intercepts Only)*

Description

Tests the null $H_0 : \alpha = 0$ that the intercepts of the test assets are jointly zero when regressed on the benchmark assets, i.e., benchmarks span the mean of the test assets. This is the F1 test of Kan & Zhou (2012).

Usage

```
span_f1(R1, R2)
```

Arguments

R1	Numeric matrix of benchmark returns, dimension $T \times K$.
R2	Numeric matrix of test-asset returns, dimension $T \times N$.

Details

Under standard assumptions (i.i.d. returns, full-rank covariances), the reference distribution is $F_{N,\,T-K-N}$. Finite-sample feasibility requires $T - K - N \geq 1$.

Value

A named list with components:

`pval` P-value for the F-statistic under the null.

`stat` F1 F-statistic.

`H0` Null hypothesis description, `"alpha = 0"`.

References

Kan R, Zhou G (2012). "Tests of Mean-Variance Spanning." *Annals of Economics and Finance*, **13**(1), 145–193.

See Also

Other Alpha Spanning Tests: span_bj(), span_gl_a(), span_grs(), span_py()

Examples

```
set.seed(123)
R1 <- matrix(rnorm(300), 100, 3)  # benchmarks: T=100, K=3
R2 <- matrix(rnorm(200), 100, 2)  # tests:      T=100, N=2
out <- span_f1(R1, R2)
out$stat; out$pval; out$H0
```

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span_f2 — *F2 Variance-Spanning Test (Slopes Only)*

Description

Tests the null $H_0 : \delta = 0$ that adding test assets does not improve the minimum-variance frontier spanned by the benchmarks (variance spanning). The statistic compares frontier-defining quantities of the augmented (benchmark + test) universe to those of the benchmark subset.

Usage

```
span_f2(R1, R2)
```

Arguments

R1	Numeric matrix of benchmark returns, dimension $T \times K$.
R2	Numeric matrix of test-asset returns, dimension $T \times N$.

Details

Under standard conditions (i.i.d. returns, full-rank covariances), the reference distribution is $F_{N,\,T-K-N+1}$. Finite-sample feasibility requires $T - K - N + 1 \geq 1$.

Value

A named list with components:

`pval` P-value for the F-statistic under the null.

`stat` F2 F-statistic.

`H0` Null hypothesis description, `"delta = 0"`.

References

Kan R, Zhou G (2012). "Tests of Mean-Variance Spanning." *Annals of Economics and Finance*, **13**(1), 145–193.

See Also

Other Variance Spanning Tests: `span_km()`

Examples

```
set.seed(123)
R1 <- matrix(rnorm(300), 100, 3)  # benchmarks: T=100, K=3
R2 <- matrix(rnorm(200), 100, 2)  # tests:      T=100, N=2
out <- span_f2(R1, R2)
out$stat; out$pval; out$H0
```

---

span_gl_a — *Gungor–Luger Alpha-Only Spanning Test (2016)*

Description

Tests the null $H_0 : \alpha = 0$ that benchmark assets span the mean (intercepts) of the test assets. Following Gungor & Luger (2016), the procedure uses a Monte Carlo (MC) test based on an $F_{\max}$ statistic with residual sign-flip simulations, yielding Least-Favorable (LMC) and Balanced (BMC) MC p-values and a three-way decision rule.

Usage

```
span_gl_a(R1, R2, control = list())
```

Arguments

`R1`	Numeric matrix of benchmark returns, dimension $T \times N$.
`R2`	Numeric matrix of test-asset returns, dimension $T \times K$.
`control`	List of options: `totsim` Number of MC simulations (default `500`). `pval_thresh` Significance level for decisions (default `0.05`). `do_trace` Logical; print progress (default `TRUE`).

Details

Accept if `pval_LMC` > `alpha`; Reject if `pval_BMC` <= `alpha`; otherwise Inconclusive. The subseries sign-flip MC approach is robust to heteroskedasticity, serial dependence, and heavy tails, making it suitable for high-dimensional settings where classical alpha tests (e.g., GRS) may suffer from size distortions.

Value

A list with components:

`pval_LMC` Least-Favorable MC p-value.

`pval_BMC` Balanced MC p-value.

`stat` Observed $F_{\max}$ statistic.

`Decisions` Decision code: `1` = Accept, `0` = Reject, `NA` = Inconclusive.

`Decisions_string` Text label: `"Accept"`, `"Reject"`, or `"Inconclusive"`.

`H0` Null hypothesis description, `"alpha = 0"`.

References

Gungor S, Luger R (2016). "Multivariate Tests of Mean-Variance Efficiency and Spanning With a Large Number of Assets and Time-Varying Covariances." *Journal of Business & Economic Statistics*, **34**(2), 161–175.

See Also

Other Alpha Spanning Tests: `span_bj()`, `span_f1()`, `span_grs()`, `span_py()`

Examples

```
set.seed(1234)
R1 <- matrix(rnorm(300), 100, 3)
R2 <- matrix(rnorm(200), 100, 2)
out <- span_gl_a(R1, R2, control = list(totsim = 100, do_trace = FALSE))
out$Decisions_string; out$pval_LMC; out$pval_BMC
```

`span_gl_ad` *Gungor–Luger Joint Mean–Variance Spanning Test (2016)*

Description

Tests the joint null $H_0: \ \alpha = 0, \ \delta = 0$ that benchmark assets span both intercepts and slopes of the test assets, allowing for heteroskedasticity, serial dependence, and time-varying covariances. Following Gungor & Luger (2016), the procedure uses a Monte Carlo (MC) test based on an $F_{\max}$ statistic with residual sign-flip simulations, yielding Least-Favorable (LMC) and Balanced (BMC) MC p-values and a three-way decision rule.

Usage

```
span_gl_ad(R1, R2, control = list())
```

Arguments

- `R1` Numeric matrix of benchmark returns, dimension $T \times N$.
- `R2` Numeric matrix of test-asset returns, dimension $T \times K$.
- `control` List of options:
  - `totsim` Number of MC simulations (default `500`).
  - `pval_thresh` Significance level for decisions (default `0.05`).
  - `do_trace` Logical; print progress (default `TRUE`).

Details

LMC/BMC follow Gungor & Luger's MC framework with residual sign-flip draws under the null. The rule is: Accept if `pval_LMC > alpha`; Reject if `pval_BMC <= alpha`; otherwise Inconclusive. This approach is robust in high-dimensional and time-varying volatility settings where classical joint spanning tests can be unreliable.

Value

A list with components:

- `pval_LMC` Least-Favorable MC p-value.
- `pval_BMC` Balanced MC p-value.
- `stat` Observed $F_{\max}$ statistic.
- `Decisions` Decision code: `1` = Accept, `0` = Reject, `NA` = Inconclusive.
- `Decisions_string` Text label: `"Accept"`, `"Reject"`, or `"Inconclusive"`.
- `H0` Null hypothesis description, `"alpha = 0 and delta = 0"`.

References

Gungor S, Luger R (2016). "Multivariate Tests of Mean-Variance Efficiency and Spanning With a Large Number of Assets and Time-Varying Covariances." *Journal of Business & Economic Statistics*, **34**(2), 161–175.

See Also

Other Joint Mean-Variance Spanning Tests: `span_hk()`

Examples

```
set.seed(123)
R1 <- matrix(rnorm(300), 100, 3)
R2 <- matrix(rnorm(200), 100, 2)
out <- span_gl_ad(R1, R2, control = list(totsim = 100, do_trace = FALSE))
out$Decisions_string; out$pval_LMC; out$pval_BMC
```

span_grs *Gibbons–Ross–Shanken (GRS) Alpha Spanning Test (1989)*

Description

Implements the GRS test of the joint null $H_0: \ \alpha = 0$ in the multivariate regression of test-asset returns on benchmark portfolios (with an intercept). The statistic assumes homoskedastic, normally distributed errors and is most reliable when T is large relative to K (benchmarks) and N (test assets).

Usage

```
span_grs(R1, R2)
```

Arguments

R1	Numeric matrix of benchmark returns, dimension $T \times K$.
R2	Numeric matrix of test-asset returns, dimension $T \times N$.

Details

Under standard conditions, the reference distribution is $F_{N,\,T-N-K}$. Finite-sample feasibility requires $T - N - K \geq 1$.

Value

A named list with components:

`pval` P-value for the F-statistic under the null.

`stat` GRS F-statistic.

`H0` Null hypothesis description, `"alpha = 0"`.

References

Gibbons MR, Ross SA, Shanken J (1989). "A Test of the Efficiency of a Given Portfolio." *Econometrica*, **57**(5), 1121–1152.

See Also

Other Alpha Spanning Tests: `span_bj()`, `span_f1()`, `span_gl_a()`, `span_py()`

Examples

```
set.seed(42)
R1 <- matrix(rnorm(300), 100, 3)  # benchmarks: T=100, K=3
R2 <- matrix(rnorm(200), 100, 2)  # tests:      T=100, N=2
out <- span_grs(R1, R2)
out$stat; out$pval; out$H0
```

---

span_hk *Huberman–Kandel Joint Mean–Variance Spanning Test (1987)*

Description

Tests the joint null $H_0: \ \alpha = 0, \ \delta = 0$ that the benchmark assets span the mean–variance frontier of the augmented (benchmark + test) universe. Following Huberman & Kandel (1987), the statistic compares the frontiers with and without the additional assets.

Usage

```
span_hk(R1, R2)
```

Arguments

R1	Numeric matrix of benchmark returns, dimension $T \times K$.
R2	Numeric matrix of test-asset returns, dimension $T \times N$.

Details

The test evaluates whether adding the test assets changes the efficient frontier implied by the benchmarks. Under standard regularity conditions, the statistic has an F reference with $(2N, \ 2(T - K - N))$ degrees of freedom. Finite-sample feasibility requires $T - K - N \geq 1$.

Value

A named list with components:

`pval` P-value for the F-statistic under the null.

`stat` F-statistic value.

`H0` Null hypothesis description, `"alpha = 0 and delta = 0"`.

References

Huberman G, Kandel S (1987). "Mean-Variance Spanning." *The Journal of Finance*, **42**(4), 873–888.

See Also

Other Joint Mean-Variance Spanning Tests: span_gl_ad()

Examples

```
set.seed(123)
R1 <- matrix(rnorm(300), 100, 3)  # benchmarks: T=100, K=3
R2 <- matrix(rnorm(200), 100, 2)  # tests:      T=100, N=2
out <- span_hk(R1, R2)
out$stat; out$pval; out$H0
```

---

span_km — *Kempf–Memmel GMVP Spanning Test*

Description

Tests whether the Global Minimum Variance Portfolio (GMVP) of the combined (benchmark + test) universe equals the GMVP of the benchmark assets alone. Following Kempf & Memmel (2006), the null assesses whether adding new assets improves the minimum-variance frontier.

Usage

```
span_km(R1, R2)
```

Arguments

R1	Numeric matrix of benchmark returns, dimension $T \times N$.
R2	Numeric matrix of test-asset returns, dimension $T \times K$.

Details

The null hypothesis H_0 is that augmenting the benchmark set with the test assets does not change the GMVP weights ($\Delta = 0$), i.e., the GMVP of the full universe coincides with that of the benchmark subset. The test is implemented via a linear restriction on coefficients in an equivalent regression representation, yielding an F-statistic.

Value

A named list with components:

`pval` P-value for the F-statistic under the null.

`stat` F-statistic value.

`H0` Null hypothesis description, "GMVP(bmk) = GMVP(full)".

References

Kempf A, Memmel C (2006). "Estimating the Global Minimum Variance Portfolio." *Schmalenbach Business Review*, **58**(4), 332–348.

See Also

Other Variance Spanning Tests: `span_f2()`

Examples

```
set.seed(123)
R1 <- matrix(rnorm(300), 100, 3)  # benchmarks: T=100, N=3
R2 <- matrix(rnorm(200), 100, 2)  # tests:      T=100, K=2
ans <- span_km(R1, R2)
ans$pval; ans$stat; ans$H0
```

---

span_py — *Pesaran–Yamagata Alpha Spanning Test (2024)*

Description

Implements the Pesaran–Yamagata test for the joint null that all intercepts are zero in a multi-factor spanning regression with possible cross-sectional dependence across test assets.

Usage

```
span_py(R1, R2)
```

Arguments

`R1`	Numeric matrix of benchmark returns, dimension $T \times K$.
`R2`	Numeric matrix of test-asset returns, dimension $T \times N$.

Details

The null hypothesis is that all intercepts are zero ($\alpha = 0$), meaning the benchmark assets span the expected returns of the test assets. The statistic adjusts for cross-sectional dependence via the residual covariance and has an asymptotic $\mathcal{N}(0, 1)$ reference under large T, N. Finite-sample safeguards require $T - K - 1 > 4$.

Value

A named list with components:

`pval` P-value under the standard normal reference distribution.

`stat` Standardized test statistic.

`H0` Null hypothesis description, `"alpha = 0"`.

References

Pesaran MH, Yamagata T (2024). "Testing for alpha in linear factor pricing models with a large number of securities." *Journal of Financial Econometrics*, **22**(2), 407–460.

See Also

Other Alpha Spanning Tests: `span_bj()`, `span_f1()`, `span_gl_a()`, `span_grs()`

Examples

```
set.seed(123)
R1 <- matrix(rnorm(300), 100, 3)  # benchmarks: T=100, K=3
R2 <- matrix(rnorm(200), 100, 2)  # tests:      T=100, N=2
out <- span_py(R1, R2)
out$pval; out$stat; out$H0
```

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