Package ‘scoringfunctions’

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Title A Collection of Scoring Functions for Assessing Point Forecasts

Description Implements multiple consistent scoring functions (Gneiting T (2011) <doi:10.1198/jasa.2011.r10138>) for assessing point forecasts and point predictions. Detailed documentation of scoring functions' properties is included for facilitating interpretation of results.

Depends R (>= 4.0.0)

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R topics documented:

`aerr_sf` *Absolute error scoring function*

Description

The function aerr_sf computes the absolute error scoring function when y materializes and x is the predictive median functional.

The absolute error scoring function is defined in Table 1 in Gneiting (2011).

Usage

```
aerr_sf(x, y)
```

Arguments

x	Predictive median functional (prediction). It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).

Details

The absolute error scoring function is defined by:

$$S(x, y) := |x - y|$$

Domain of function:

$$x \in \mathsf{R}$$

$$y \in \mathsf{R}$$

Range of function:

$$S(x, y) \geq 0, \forall x, y \in \mathsf{R}$$

Value

Vector of absolute errors.

Note

For details on the absolute error scoring function, see Gneiting (2011).

The median functional is the median of the probability distribution F of y (Gneiting 2011).

The absolute error scoring function is negatively oriented (i.e. the smaller, the better).

The absolute error scoring function is strictly consistent for the median functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which the first moment exists and is finite (Thomson 1979, Saerens 2000, Gneiting 2011).

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Saerens M (2000) Building cost functions minimizing to some summary statistics. *IEEE Transactions on Neural Networks* **11(6)**:1263–1271. doi:10.1109/72.883416.

Thomson W (1979) Eliciting production possibilities from a well-informed manager. *Journal of Economic Theory* **20(3)**:360–380. doi:10.1016/00220531(79)900425.

Examples

```
# Compute the absolute error scoring function.

df <- data.frame(
    y = rep(x = 0, times = 5),
    x = -2:2
)

df$absolute_error <- aerr_sf(x = df$x, y = df$y)

print(df)
```

---

aperr_sf *Absolute percentage error scoring function*

Description

The function aperr_sf computes the absolute percentage error scoring function when y materializes and x is the predictive $\mathrm{med}^{(-1)}(F)$ functional.

The absolute percentage error scoring function is defined in Table 1 in Gneiting (2011).

Usage

```
aperr_sf(x, y)
```

Arguments

x Predictive $\text{med}^{(-1)}(F)$ functional (prediction). It can be a vector of length n (must have the same length as y).

y Realization (true value) of process. It can be a vector of length n (must have the same length as x).

Details

The absolute percentage error scoring function is defined by:

$$S(x, y) := |(x - y)/y|$$

Domain of function:

$$x > 0$$

$$y > 0$$

Range of function:

$$S(x, y) \geq 0, \forall x, y > 0$$

Value

Vector of absolute percentage errors.

Note

For details on the absolute percentage error scoring function, see Gneiting (2011).

The β-median functional, $\text{med}^{(\beta)}(F)$ is the median of a probability distribution whose density is proportional to $y^\beta f(y)$, where f is the density of the probability distribution F of y (Gneiting 2011).

The absolute percentage error scoring function is negatively oriented (i.e. the smaller, the better).

The absolute percentage error scoring function is strictly consistent for the $\text{med}^{(-1)}(F)$ functional relative to the family $\mathbb{F}$ of potential probability distributions (whose densities are proportional to $y^{-1} f(y)$, where f is the density of the probability distribution F for the future y) for which the first moment exists and is finite (see Theorems 5 and 9 in Gneiting 2011).

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Examples

```
# Compute the absolute percentage error scoring function.

df <- data.frame(
    y = rep(x = 2, times = 3),
    x = 1:3
)

df$absolute_percentage_error <- aperr_sf(x = df$x, y = df$y)

print(df)
```

---

bmedian_sf *β-median scoring function*

Description

The function bmedian_sf computes the β-median scoring function when y materializes and x is the predictive $\text{med}^{(\beta)}(F)$ functional.

The β-median scoring function is defined in eq. (4) in Gneiting (2011).

Usage

```
bmedian_sf(x, y, b)
```

Arguments

x	Predictive $\text{med}^{(\beta)}(F)$ functional (prediction). It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).
b	It can be a vector of length n (must have the same length as y).

Details

The β-median scoring function is defined by:

$$S(x, y, b) := |1 - (y/x)^b|$$

Domain of function:

$$x > 0$$

$$y > 0$$

$$b \neq 0$$

Range of function:

$$S(x, y, b) \geq 0, \forall x, y > 0, b \neq 0$$

Value

Vector of β-median losses.

Note

For details on the β-median scoring function, see Gneiting (2011).

The β-median functional, $\mathrm{med}^{(\beta)}(F)$ is the median of a probability distribution whose density is proportional to $y^\beta f(y)$, where f is the density of the probability distribution F of y (Gneiting 2011).

The β-median scoring function is negatively oriented (i.e. the smaller, the better).

The β-median scoring function is strictly consistent for the $\mathrm{med}^{(\beta)}(F)$ functional relative to the family $\mathbb{F}$ of potential probability distributions (whose densities are proportional to $y^\beta f(y)$, where f is the density of the probability distribution F for the future y) (see Theorems 5 and 9 in Gneiting 2011).

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Examples

```
# Compute the bmedian scoring function.

df <- data.frame(
    y = rep(x = 2, times = 3),
    x = 1:3,
    b = c(-1, 1, 2)
)

df$bmedian_error <- bmedian_sf(x = df$x, y = df$y, b = df$b)

print(df)
```

bregman1_sf *Bregman scoring function (type 1)*

Description

The function bregman1_sf computes the Bregman scoring function when y materializes and x is the predictive mean functional.

The Bregman scoring function is defined by eq. (18) in Gneiting (2011) and the form implemented here for $\phi(x) = |x|^a$ is defined by eq. (19) in Gneiting (2011).

Usage

```
bregman1_sf(x, y, a)
```

Arguments

x	Predictive mean functional (prediction). It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).
a	It can be a vector of length n (must have the same length as y).

Details

The Bregman scoring function (type 1) is defined by:

$$S(x, y, a) := |y|^a - |x|^a - a\mathrm{sign}(x)|x|^{a-1}(y - x)$$

Domain of function:

$$x \in \mathsf{R}$$

$$y \in \mathsf{R}$$

$$a > 1$$

Range of function:

$$S(x, y, a) \geq 0, \forall x, y \in \mathsf{R}, a > 1$$

Value

Vector of Bregman losses.

Note

The implemented function is denoted as type 1 since it corresponds to a specific type of $\phi(x)$ of the general form of the Bregman scoring function defined by eq. (18) in Gneiting (2011).

For details on the Bregman scoring function, see Savage 1971, Banerjee et al. (2005) and Gneiting (2011).

The mean functional is the mean $E_F[Y]$ of the probability distribution F of y (Gneiting 2011).

The Bregman scoring function is negatively oriented (i.e. the smaller, the better).

The herein implemented Bregman scoring function is strictly consistent for the mean functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which $E_F[Y]$ and $E_F[|Y|^a]$ exist and are finite (Savage 1971, Gneiting 2011).

References

Banerjee A, Guo X, Wang H (2005) On the optimality of conditional expectation as a Bregman predictor. *IEEE Transactions on Information Theory* **51(7)**:2664–2669. doi:10.1109/TIT.2005.850145.

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Savage LJ (1971) Elicitation of personal probabilities and expectations. *Journal of the American Statistical Association* **66(337)**:783–810. doi:10.1080/01621459.1971.10482346.

Examples

```
# Compute the Bregman scoring function (type 1).

df <- data.frame(
    y = rep(x = 0, times = 7),
    x = c(-3, -2, -1, 0, 1, 2, 3),
    a = rep(x = 3, times = 7)
)

df$bregman1_penalty <- bregman1_sf(x = df$x, y = df$y, a = df$a)

print(df)

# Equivalence of Bregman scoring function (type 1) and squared error scoring
# function, when a = 2.

set.seed(12345)

n <- 100

x <- runif(n, -20, 20)
y <- runif(n, -20, 20)
a <- rep(x = 2, times = n)

u <- bregman1_sf(x = x, y = y, a = a)

v <- serr_sf(x = x, y = y)
```

```
max(abs(u - v)) # values are slightly higher than 0 due to rounding error
min(abs(u - v))
```

---

bregman2_sf *Bregman scoring function (type 2, Patton scoring function)*

Description

The function bregman2_sf computes the Bregman scoring function when y materializes and x is the predictive mean functional.

The Bregman scoring function is defined by eq. (18) in Gneiting (2011) and the form implemented here for $\phi(x) = \frac{1}{b(b-1)} x^b$, $b \in \mathsf{R} \setminus \{0, 1\}$ is defined by eq. (20) in Gneiting (2011).

Usage

```
bregman2_sf(x, y, b)
```

Arguments

x	Predictive mean functional (prediction). It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).
b	It can be a vector of length n (must have the same length as y).

Details

The Bregman scoring function (type 2) is defined by:

$$S(x, y, b) := \frac{1}{b(b-1)} (y^b - x^b) - \frac{1}{b-1} x^{b-1} (y - x)$$

Domain of function:

$$x > 0$$

$$y > 0$$

$$b \in \mathsf{R} \setminus \{0, 1\}$$

Range of function:

$$S(x, y, b) \geq 0, \forall x, y > 0, b \in \mathsf{R} \setminus \{0, 1\}$$

Value

Vector of Bregman losses.

Note

The implemented function is denoted as type 2 since it corresponds to a specific type of $\phi(x)$ of the general form of the Bregman scoring function defined by eq. (18) in Gneiting (2011).

For details on the Bregman scoring function, see Savage 1971, Banerjee et al. (2005) and Gneiting (2011). For details on the specific form implemented here, see Patton (2011).

The mean functional is the mean $E_F[Y]$ of the probability distribution F of y (Gneiting 2011).

The Bregman scoring function is negatively oriented (i.e. the smaller, the better).

The herein implemented Bregman scoring function is strictly consistent for the mean functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which $E_F[Y]$ and $E_F[\frac{1}{b(b-1)}Y^b]$ exist and are finite (Savage 1971, Gneiting 2011).

References

Banerjee A, Guo X, Wang H (2005) On the optimality of conditional expectation as a Bregman predictor. *IEEE Transactions on Information Theory* **51(7)**:2664–2669. doi:10.1109/TIT.2005.850145.

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Patton AJ (2011) Volatility forecast comparison using imperfect volatility proxies. *Journal of Econometrics* **160(1)**:246–256. doi:10.1016/j.jeconom.2010.03.034.

Savage LJ (1971) Elicitation of personal probabilities and expectations. *Journal of the American Statistical Association* **66(337)**:783–810. doi:10.1080/01621459.1971.10482346.

Examples

```
# Compute the Bregman scoring function (type 2).

df <- data.frame(
    y = rep(x = 2, times = 6),
    x = rep(x = 1:3, times = 2),
    b = rep(x = c(-3, 3), each = 3)
)

df$bregman2_penalty <- bregman2_sf(x = df$x, y = df$y, b = df$b)

print(df)

# The Bregman scoring function (type 2) is half the squared error scoring
# function, when b = 2.

df <- data.frame(
    y = rep(x = 5.5, times = 10),
    x = 1:10,
    b = rep(x = 2, times = 10)
```

```
)

df$bregman2_penalty <- bregman2_sf(x = df$x, y = df$y, b = df$b)

df$squared_error <- serr_sf(x = df$x, y = df$y)

df$ratio <- df$bregman2_penalty/df$squared_error

print(df)


# When a = b > 1 the Bregman scoring function (type 2) coincides with the
# Bregman scoring function (type 1) up to a multiplicative constant.

df <- data.frame(
    y = rep(x = 5.5, times = 10),
    x = 1:10,
    b = rep(x = c(3, 4), each = 5)
)

df$bregman2_penalty <- bregman2_sf(x = df$x, y = df$y, b = df$b)

df$bregman1_penalty <- bregman1_sf(x = df$x, y = df$y, a = df$b)

df$ratio <- df$bregman2_penalty/df$bregman1_penalty

print(df)
```

`bregman3_sf` *Bregman scoring function (type 3, QLIKE scoring function)*

Description

The function bregman3_sf computes the Bregman scoring function when y materializes and x is the predictive mean functional.

The Bregman scoring function is defined by eq. (18) in Gneiting (2011) and the form implemented here for $\phi(x) = -\log(x)$ is defined by eq. (20) in Gneiting (2011).

Usage

```
bregman3_sf(x, y)
```

Arguments

x	Predictive mean functional (prediction). It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).

Details

The Bregman scoring function (type 3) is defined by:

$$S(x, y) := (y/x) - \log(y/x) - 1$$

Domain of function:

$$x > 0$$

$$y > 0$$

Range of function:

$$S(x, y) \geq 0, \forall x, y > 0$$

Value

Vector of Bregman losses.

Note

The implemented function is denoted as type 3 since it corresponds to a specific type of $\phi(x)$ of the general form of the Bregman scoring function defined by eq. (18) in Gneiting (2011).

For details on the Bregman scoring function, see Savage 1971, Banerjee et al. (2005) and Gneiting (2011). For details on the specific form implemented here, see the QLIKE scoring function in Patton (2011).

The mean functional is the mean $E_F[Y]$ of the probability distribution F of y (Gneiting 2011).

The Bregman scoring function is negatively oriented (i.e. the smaller, the better).

The herein implemented Bregman scoring function is strictly consistent for the mean functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which $E_F[Y]$ and $E_F[\log(Y)]$ exist and are finite (Savage 1971, Gneiting 2011).

References

Banerjee A, Guo X, Wang H (2005) On the optimality of conditional expectation as a Bregman predictor. *IEEE Transactions on Information Theory* **51(7)**:2664–2669. doi:10.1109/TIT.2005.850145.

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Patton AJ (2011) Volatility forecast comparison using imperfect volatility proxies. *Journal of Econometrics* **160(1)**:246–256. doi:10.1016/j.jeconom.2010.03.034.

Savage LJ (1971) Elicitation of personal probabilities and expectations. *Journal of the American Statistical Association* **66(337)**:783–810. doi:10.1080/01621459.1971.10482346.

Examples

```
# Compute the Bregman scoring function (type 3, QLIKE scoring function).

df <- data.frame(
    y = rep(x = 2, times = 3),
    x = 1:3
)

df$bregman3_penalty <- bregman3_sf(x = df$x, y = df$y)

print(df)
```

`bregman4_sf` *Bregman scoring function (type 4, Patton scoring function)*

Description

The function bregman4_sf computes the Bregman scoring function when y materializes and x is the predictive mean functional.

The Bregman scoring function is defined by eq. (18) in Gneiting (2011) and the form implemented here for $\phi(x) = x \log(x)$ is defined by eq. (20) in Gneiting (2011).

Usage

```
bregman4_sf(x, y)
```

Arguments

x	Predictive mean functional (prediction). It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).

Details

The Bregman scoring function (type 4) is defined by:

$$S(x, y) := y \log(y/x) - y + x$$

Domain of function:

$$x > 0$$

$$y > 0$$

Range of function:

$$S(x, y) \geq 0, \forall x, y > 0$$

Value

Vector of Bregman losses.

Note

The implemented function is denoted as type 4 since it corresponds to a specific type of $\phi(x)$ of the general form of the Bregman scoring function defined by eq. (18) in Gneiting (2011).

For details on the Bregman scoring function, see Savage 1971, Banerjee et al. (2005) and Gneiting (2011). For details on the specific form implemented here, see Patton (2011).

The mean functional is the mean $E_F[Y]$ of the probability distribution F of y (Gneiting 2011).

The Bregman scoring function is negatively oriented (i.e. the smaller, the better).

The herein implemented Bregman scoring function is strictly consistent for the mean functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which $E_F[Y]$ and $E_F[Y \log(Y)]$ exist and are finite (Savage 1971, Gneiting 2011).

References

Banerjee A, Guo X, Wang H (2005) On the optimality of conditional expectation as a Bregman predictor. *IEEE Transactions on Information Theory* **51(7)**:2664–2669. doi:10.1109/TIT.2005.850145.

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Patton AJ (2011) Volatility forecast comparison using imperfect volatility proxies. *Journal of Econometrics* **160(1)**:246–256. doi:10.1016/j.jeconom.2010.03.034.

Savage LJ (1971) Elicitation of personal probabilities and expectations. *Journal of the American Statistical Association* **66(337)**:783–810. doi:10.1080/01621459.1971.10482346.

Examples

```
# Compute the Bregman scoring function (type 4).

df <- data.frame(
    y = rep(x = 2, times = 3),
    x = 1:3
)

df$bregman4_penalty <- bregman4_sf(x = df$x, y = df$y)

print(df)
```

capping_function *Capping function*

Description

The function capping_function computes the value of the capping function, defined in Taggart (2022), p.205.

It is used by the generalized Huber loss function among others (see Taggart 2022).

Usage

```
capping_function(t, a, b)
```

Arguments

t	It can be a vector of length n.
a	It can be a vector of length n (must have the same length as t).
b	It can be a vector of length n (must have the same length as t).

Details

The capping function $\kappa_{a,b}(t)$ is defined by:

$$\kappa_{a,b}(t) := \max\{\min\{t, b\}, -a\}$$

Domain of function:

$$t \in \mathsf{R}$$

$$a \geq 0$$

$$b \geq 0$$

Value

Vector of values of the capping function.

Note

For the definition of the capping function, see Taggart (2022), p.205.

References

Taggart RJ (2022) Point forecasting and forecast evaluation with generalized Huber loss. *Electronic Journal of Statistics* **16**:201–231. doi:10.1214/21EJS1957.

Examples

```
# Compute the capping function.

df <- data.frame(
    t = c(1, -1, 1, -1, 1, -1, 1, -1, 1, 1, 2.5, 2.5, 3.5, 3.5),
    a = c(0, 0, 0, 0, Inf, Inf, Inf, Inf, 2, 3, 2, 3, 2, 3),
    b = c(0, 0, Inf, Inf, 0, 0, Inf, Inf, 3, 2, 3, 2, 3, 2)
)

df$cf <- capping_function(t = df$t, a = df$a, b = df$b)

print(df)
```

`expectile_sf` *Asymmetric piecewise quadratic scoring function (expectile scoring function)*

Description

The function expectile_sf computes the asymmetric piecewise quadratic scoring function (expectile scoring function) at a specific level p, when y materializes and x is the predictive expectile at level p.

The asymmetric piecewise quadratic scoring function is defined by eq. (27) in Gneiting (2011).

Usage

```
expectile_sf(x, y, p)
```

Arguments

x	Predictive expectile (prediction) at level p. It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).
p	It can be a vector of length n (must have the same length as y).

Details

The asymmetric piecewise quadratic scoring function is defined by:

$$S(x, y, p) := |1(x \geq y) - p|(x - y)^2$$

Domain of function:

$$x \in \mathsf{R}$$

$$y \in \mathsf{R}$$

$$0 < p < 1$$

Range of function:

$$S(x, y, p) \geq 0, \forall x, y \in \mathsf{R}, p \in (0, 1)$$

Value

Vector of expectile losses.

Note

For the definition of expectiles, see Newey and Powell (1987).

The asymmetric piecewise quadratic scoring function is negatively oriented (i.e. the smaller, the better).

The asymmetric piecewise quadratic scoring function is strictly consistent for the p-expectile functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which $E_F[Y^2]$ exists and is finite (Gneiting 2011).

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Newey WK, Powell JL (1987) Asymmetric least squares estimation and testing. *Econometrica* **55(4)**:819–847. doi:10.2307/1911031.

Examples

```
# Compute the asymmetric piecewise quadratic scoring function (expectile scoring
# function).

df <- data.frame(
    y = rep(x = 0, times = 6),
    x = c(2, 2, -2, -2, 0, 0),
    p = rep(x = c(0.05, 0.95), times = 3)
)

df$expectile_penalty <- expectile_sf(x = df$x, y = df$y, p = df$p)

print(df)

# The asymmetric piecewise quadratic scoring function (expectile scoring
# function) at level p = 0.5 is half the squared error scoring function.

df <- data.frame(
    y = rep(x = 0, times = 3),
```

```
    x = c(-2, 0, 2),
    p = rep(x = c(0.5), times = 3)
)

df$expectile_penalty <- expectile_sf(x = df$x, y = df$y, p = df$p)

df$squared_error <- serr_sf(x = df$x, y = df$y)

print(df)
```

`ghuber_sf` *Generalized Huber scoring function*

Description

The function ghuber_sf computes the generalized Huber scoring function at a specific level p and parameters a and b, when y materializes and x is the predictive Huber functional at level p.

The generalized Huber scoring function is defined by eq. (4.7) in Taggart (2022) for $\phi(t) = t^2$.

Usage

```
ghuber_sf(x, y, p, a, b)
```

Arguments

x	Predictive Huber functional (prediction) at level p. It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).
p	It can be a vector of length n (must have the same length as y).
a	It can be a vector of length n (must have the same length as y).
b	It can be a vector of length n (must have the same length as y).

Details

The generalized Huber scoring function is defined by:

$$S(x, y, p, a, b) := |1(x \geq y) - p|(y^2 - (\kappa_{a,b}(x - y) + y)^2 + 2x\kappa_{a,b}(x - y))$$

where $\kappa_{a,b}(t)$ is the capping function defined by:

$$\kappa_{a,b}(t) := \max\{\min\{t, b\}, -a\}$$

Domain of function:

$$x \in \mathsf{R}$$

$$y \in \mathsf{R}$$

$$0 < p < 1$$

$$a > 0$$

$$b > 0$$

Range of function:

$$S(x, y, p, a, b) \geq 0, \forall x, y \in \mathsf{R}, p \in (0, 1), a, b > 0$$

Value

Vector of generalized Huber losses.

Note

For the definition of Huber functionals, see definition 3.3 in Taggart (2022). The value of eq. (4.7) is twice the value of the equation in definition 4.2 in Taggart (2002).

The generalized Huber scoring function is negatively oriented (i.e. the smaller, the better).

The generalized Huber scoring function is strictly consistent for the p-Huber functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which $E_F[Y^2 - (Y - a)^2]$ and $E_F[Y^2 - (Y + b)^2]$ exist and are finite (Taggart 2022).

References

Taggart RJ (2022) Point forecasting and forecast evaluation with generalized Huber loss. *Electronic Journal of Statistics* **16**:201–231. doi:10.1214/21EJS1957.

Examples

```
# Compute the generalized Huber scoring function.

set.seed(12345)

n <- 10

df <- data.frame(
    x = runif(n, -2, 2),
    y = runif(n, -2, 2),
    p = runif(n, 0, 1),
    a = runif(n, 0, 1),
    b = runif(n, 0, 1)
)
```

```
df$ghuber_penalty <- ghuber_sf(x = df$x, y = df$y, p = df$p, a = df$a, b = df$b)

print(df)

# Equivalence of the generalized Huber scoring function and the asymmetric
# piecewise quadratic scoring function (expectile scoring function), when
# a = Inf and b = Inf.

set.seed(12345)

n <- 100

x <- runif(n, -20, 20)
y <- runif(n, -20, 20)
p <- runif(n, 0, 1)
a <- rep(x = Inf, times = n)
b <- rep(x = Inf, times = n)

u <- ghuber_sf(x = x, y = y, p = p, a = a, b = b)
v <- expectile_sf(x = x, y = y, p = p)

max(abs(u - v)) # values are slightly higher than 0 due to rounding error
min(abs(u - v))

# Equivalence of the generalized Huber scoring function and the Huber scoring
# function when p = 1/2 and a = b.

set.seed(12345)

n <- 100

x <- runif(n, -20, 20)
y <- runif(n, -20, 20)
p <- rep(x = 1/2, times = n)
a <- runif(n, 0, 20)

u <- ghuber_sf(x = x, y = y, p = p, a = a, b = a)
v <- huber_sf(x = x, y = y, a = a)

max(abs(u - v)) # values are slightly higher than 0 due to rounding error
min(abs(u - v))
```

`gpl1_sf` *Generalized piecewise linear scoring function (type 1)*

Description

The function gpl1_sf computes the generalized piecewise linear scoring function at a specific level p for $g(x) = x^b/|b|$, $b > 0$, when y materializes and x is the predictive quantile at level p.

The generalized piecewise linear scoring function is defined by eq. (25) in Gneiting (2011) and the form implemented here for the specific $g(x)$ is defined by eq. (26) in Gneiting (2011).

Usage

```
gpl1_sf(x, y, p, b)
```

Arguments

x	Predictive quantile (prediction) at level p. It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).
p	It can be a vector of length n (must have the same length as y).
b	It can be a vector of length n (must have the same length as y).

Details

The generalized piecewise linear scoring function (type 1) is defined by:

$$S(x, y, p, b) := (1/|b|)(1(x \geq y) - p)(x^b - y^b)$$

Domain of function:

$$x > 0$$

$$y > 0$$

$$0 < p < 1$$

$$b > 0$$

Range of function:

$$S(x, y, p, b) \geq 0, \forall x, y > 0, p \in (0, 1), b > 0$$

Value

Vector of generalized piecewise linear losses.

Note

The implemented function is denoted as type 1 since it corresponds to a specific type of $g(x)$ of the general form of the generalized piecewise linear scoring function defined by eq. (25) in Gneiting (2011).

For the definition of quantiles, see Koenker and Bassett Jr (1978).

The generalized piecewise linear scoring function is negatively oriented (i.e. the smaller, the better).

The herein implemented generalized piecewise linear scoring function is strictly consistent for the p-quantile functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which $E_F[Y^b]$ exists and is finite (Thomson 1979, Saerens 2000, Gneiting 2011).

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Koenker R, Bassett Jr G (1978) Regression quantiles. *Econometrica* **46(1)**:33–50. doi:10.2307/1913643.

Saerens M (2000) Building cost functions minimizing to some summary statistics. *IEEE Transactions on Neural Networks* **11(6)**:1263–1271. doi:10.1109/72.883416.

Thomson W (1979) Eliciting production possibilities from a well-informed manager. *Journal of Economic Theory* **20(3)**:360–380. doi:10.1016/00220531(79)900425.

Examples

```
# Compute the generalized piecewise linear scoring function (type 1).

df <- data.frame(
    y = rep(x = 2, times = 6),
    x = c(1, 2, 3, 1, 2, 3),
    p = c(rep(x = 0.05, times = 3), rep(x = 0.95, times = 3)),
    b = rep(x = 2, times = 6)
)

df$gpl1_penalty <- gpl1_sf(x = df$x, y = df$y, p = df$p, b = df$b)

print(df)

# Equivalence of generalized piecewise linear scoring function (type 1) and
# asymmetric piecewise linear scoring function (quantile scoring function), when
# b = 1.

set.seed(12345)

n <- 100

x <- runif(n, 0, 20)
y <- runif(n, 0, 20)
p <- runif(n, 0, 1)
b <- rep(x = 1, times = n)

u <- gpl1_sf(x = x, y = y, p = p, b = b)
v <- quantile_sf(x = x, y = y, p = p)

max(abs(u - v))

# Equivalence of generalized piecewise linear scoring function (type 1) and
# MAE-SD scoring function, when p = 1/2 and b = 1/2.

set.seed(12345)

n <- 100
```

```
x <- runif(n, 0, 20)
y <- runif(n, 0, 20)
p <- rep(x = 0.5, times = n)
b <- rep(x = 1/2, times = n)

u <- gpl1_sf(x = x, y = y, p = p, b = b)
v <- maesd_sf(x = x, y = y)

max(abs(u - v))
```

`gpl2_sf` *Generalized piecewise linear scoring function (type 2)*

Description

The function gpl2_sf computes the generalized piecewise linear scoring function at a specific level p for $g(x) = \log(x)$, when y materializes and x is the predictive quantile at level p.

The generalized piecewise linear scoring function is negatively oriented (i.e. the smaller, the better).

The generalized piecewise linear scoring function is defined by eq. (25) in Gneiting (2011) and the form implemented here for the specific $g(x)$ is defined by eq. (26) in Gneiting (2011) for $b = 0$.

Usage

```
gpl2_sf(x, y, p)
```

Arguments

`x`	Predictive quantile (prediction) at level p. It can be a vector of length n (must have the same length as y).
`y`	Realization (true value) of process. It can be a vector of length n (must have the same length as x).
`p`	It can be a vector of length n (must have the same length as y).

Details

The generalized piecewise linear scoring function (type 2) is defined by:

$$S(x, y, p) := (1(x \geq y) - p) \log(x/y)$$

Domain of function:

$$x > 0$$

$$y > 0$$

$$0 < p < 1$$

Range of function:

$$S(x, y, p) \geq 0, \forall x, y > 0, p \in (0, 1)$$

Value

Vector of generalized piecewise linear losses.

Note

The implemented function is denoted as type 2 since it corresponds to a specific type of $g(x)$ of the general form of the generalized piecewise linear scoring function defined by eq. (25) in Gneiting (2011).

For the definition of quantiles, see Koenker and Bassett Jr (1978).

The herein implemented generalized piecewise linear scoring function is strictly consistent for the p-quantile functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which $E_F[\log(Y)]$ exists and is finite (Thomson 1979, Saerens 2000, Gneiting 2011).

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Koenker R, Bassett Jr G (1978) Regression quantiles. *Econometrica* **46(1)**:33–50. doi:10.2307/1913643.

Saerens M (2000) Building cost functions minimizing to some summary statistics. *IEEE Transactions on Neural Networks* **11(6)**:1263–1271. doi:10.1109/72.883416.

Thomson W (1979) Eliciting production possibilities from a well-informed manager. *Journal of Economic Theory* **20(3)**:360–380. doi:10.1016/00220531(79)900425.

Examples

```
# Compute the generalized piecewise linear scoring function (type 2).

df <- data.frame(
    y = rep(x = 2, times = 6),
    x = c(1, 2, 3, 1, 2, 3),
    p = c(rep(x = 0.05, times = 3), rep(x = 0.95, times = 3))
)

df$gpl2_penalty <- gpl2_sf(x = df$x, y = df$y, p = df$p)

print(df)

# The generalized piecewise linear scoring function (type 2) is half the MAE-LOG
# scoring function.
```

```
df <- data.frame(
    y = rep(x = 5.5, times = 10),
    x = 1:10,
    p = rep(x = 0.5, times = 10)
)

df$gpl2_penalty <- gpl2_sf(x = df$x, y = df$y, p = df$p)

df$mae_log_penalty <- maelog_sf(x = df$x, y = df$y)

df$ratio <- df$gpl2_penalty/df$mae_log_penalty

print(df)
```

`huber_sf` *Huber scoring function*

Description

The function huber_sf computes the Huber scoring function with parameter a, when y materializes and x is the predictive Huber mean.

The Huber scoring function is defined in Huber (1964).

Usage

```
huber_sf(x, y, a)
```

Arguments

x	Predictive Huber mean (prediction). It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).
a	It can be a vector of length n (must have the same length as y).

Details

The Huber scoring function is defined by:

$$S(x, y, a) := \begin{cases} \frac{1}{2}(x-y)^2, & |x-y| \leq a \\ a|x-y| - \frac{1}{2}a^2, & |x-y| > a \end{cases}$$

Domain of function:

$$x \in \mathsf{R}$$

$$y \in \mathsf{R}$$

$$a > 0$$

Range of function:

$$S(x, y, a) \geq 0, \forall x, y \in \mathsf{R}, a > 0$$

Value

Vector of Huber losses.

Note

For the definition of Huber mean, see Taggart (2022).

The Huber scoring function is negatively oriented (i.e. the smaller, the better).

The Huber scoring function is strictly consistent for the Huber mean relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which $E_F[Y^2 - (Y - a)^2]$ and $E_F[Y^2 - (Y + a)^2]$ exist and are finite (Taggart 2022).

References

Huber PJ (1964) Robust Estimation of a Location Parameter. *Annals of Mathematical Statistics* **35(1)**:73–101. doi:10.1214/aoms/1177703732.

Taggart RJ (2022) Point forecasting and forecast evaluation with generalized Huber loss. *Electronic Journal of Statistics* **16**:201–231. doi:10.1214/21EJS1957.

Examples

```
# Compute the Huber scoring function.

df <- data.frame(
    x = c(-3, -2, -1, 0, 1, 2, 3),
    y = c(0, 0, 0, 0, 0, 0, 0),
    a = c(2.7, 2.5, 0.6, 0.7, 0.9, 1.2, 5)
)

df$huber_penalty <- huber_sf(x = df$x, y = df$y, a = df$a)

print(df)
```

maelog_sf *MAE-LOG scoring function*

Description

The function maelog_sf computes the MAE-LOG scoring function when y materializes and x is the predictive median functional.

The MAE-LOG scoring function is defined by eq. (11) in Patton (2011).

Usage

```
maelog_sf(x, y)
```

Arguments

x	Predictive median functional (prediction). It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).

Details

The MAE-LOG scoring function is defined by:

$$S(x, y) := |\log(x/y)|$$

Domain of function:

$$x > 0$$

$$y > 0$$

Range of function:

$$S(x, y) \geq 0, \forall x, y > 0$$

Value

Vector of MAE-LOG losses.

Note

For details on the MAE-LOG scoring function, see Gneiting (2011) and Patton (2011).

The median functional is the median of the probability distribution F of y (Gneiting 2011).

The MAE-LOG scoring function is negatively oriented (i.e. the smaller, the better).

The MAE-LOG scoring function is strictly consistent for the median functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which $E_F[\log(Y)]$ exists and is finite (Thomson 1979, Saerens 2000, Gneiting 2011).

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Patton AJ (2011) Volatility forecast comparison using imperfect volatility proxies. *Journal of Econometrics* **160(1)**:246–256. doi:10.1016/j.jeconom.2010.03.034.

Saerens M (2000) Building cost functions minimizing to some summary statistics. *IEEE Transactions on Neural Networks* **11(6)**:1263–1271. doi:10.1109/72.883416.

Thomson W (1979) Eliciting production possibilities from a well-informed manager. *Journal of Economic Theory* **20(3)**:360–380. doi:10.1016/00220531(79)900425.

Examples

```
# Compute the MAE-LOG scoring function.

df <- data.frame(
    y = rep(x = 2, times = 3),
    x = 1:3
)

df$mae_log_penalty <- maelog_sf(x = df$x, y = df$y)

print(df)
```

---

maesd_sf *MAE-SD scoring function*

Description

The function maesd_sf computes the MAE-SD scoring function when y materializes and x is the predictive median functional.

The MAE-SD scoring function is defined by eq. (12) in Patton (2011).

Usage

```
maesd_sf(x, y)
```

Arguments

x Predictive median functional (prediction). It can be a vector of length n (must have the same length as y).

y Realization (true value) of process. It can be a vector of length n (must have the same length as x).

Details

The MAE-SD scoring function is defined by:

$$S(x, y) := |x^{1/2} - y^{1/2}|$$

Domain of function:

$$x > 0$$

$$y > 0$$

Range of function:

$$S(x, y) \geq 0, \forall x, y > 0$$

Value

Vector of MAE-SD losses.

Note

For details on the MAE-SD scoring function, see Gneiting (2011) and Patton (2011).

The median functional is the median of the probability distribution F of y (Gneiting 2011).

The MAE-SD scoring function is negatively oriented (i.e. the smaller, the better).

The MAE-SD scoring function is strictly consistent for the median functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which $E_F[Y^{1/2}]$ exists and is finite (Thomson 1979, Saerens 2000, Gneiting 2011).

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Patton AJ (2011) Volatility forecast comparison using imperfect volatility proxies. *Journal of Econometrics* **160(1)**:246–256. doi:10.1016/j.jeconom.2010.03.034.

Saerens M (2000) Building cost functions minimizing to some summary statistics. *IEEE Transactions on Neural Networks* **11(6)**:1263–1271. doi:10.1109/72.883416.

Thomson W (1979) Eliciting production possibilities from a well-informed manager. *Journal of Economic Theory* **20(3)**:360–380. doi:10.1016/00220531(79)900425.

Examples

```
# Compute the MAE-SD scoring function.

df <- data.frame(
    y = rep(x = 2, times = 3),
    x = 1:3
)

df$mae_sd_penalty <- maesd_sf(x = df$x, y = df$y)

print(df)
```

`obsweighted_sf` *Observation-weighted scoring function*

Description

The function obsweighted_sf computes the observation-weighted scoring function when y materializes and x is the predictive $\frac{\mathrm{E}_F[Y^2]}{\mathrm{E}_F[Y]}$ functional.

The observation-weighted scoring function is defined in p. 752 in Gneiting (2011).

Usage

```
obsweighted_sf(x, y)
```

Arguments

x	Predictive $\frac{\mathrm{E}_F[Y^2]}{\mathrm{E}_F[Y]}$ functional (prediction). It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).

Details

The observation-weighted scoring function is defined by:

$$S(x, y) := y(x - y)^2$$

Domain of function:

$$x > 0$$

$$y > 0$$

Range of function:

$$S(x, y) \geq 0, \forall x, y > 0$$

Value

Vector of observation-weighted errors.

Note

For details on the observation-weighted scoring function, see Gneiting (2011).

The observation-weighted scoring function is negatively oriented (i.e. the smaller, the better).

The observation-weighted scoring function is strictly consistent for the $\frac{\mathrm{E}_F[Y^2]}{\mathrm{E}_F[Y]}$ functional.

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Examples

```
# Compute the observation-weighted scoring function.

df <- data.frame(
    y = rep(x = 2, times = 3),
    x = 1:3
)

df$squared_relative_error <- obsweighted_sf(x = df$x, y = df$y)

print(df)
```

---

quantile_sf   *Asymmetric piecewise linear scoring function (quantile scoring function)*

Description

The function quantile_sf computes the asymmetric piecewise linear scoring function (quantile scoring function) at a specific level p, when y materializes and x is the predictive quantile at level p.

The asymmetric piecewise linear scoring function is defined by eq. (24) in Gneiting (2011).

Usage

```
quantile_sf(x, y, p)
```

Arguments

x	Predictive quantile (prediction) at level p. It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).
p	It can be a vector of length n (must have the same length as y).

Details

The assymetric piecewise linear scoring function is defined by:

$$S(x, y, p) := (1(x \geq y) - p)(x - y)$$

Domain of function:

$$x \in \mathsf{R}$$

$$y \in \mathsf{R}$$

$$0 < p < 1$$

Range of function:

$$S(x, y, p) \geq 0, \forall x, y \in \mathsf{R}, p \in (0, 1)$$

Value

Vector of quantile losses.

Note

For the definition of quantiles, see Koenker and Bassett Jr (1978).

The asymmetric piecewise linear scoring function is negatively oriented (i.e. the smaller, the better).

The asymmetric piecewise linear scoring function is strictly consistent for the p-quantile functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which $E_F[Y]$ exists and is finite (Thomson 1979, Saerens 2000, Gneiting 2011).

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Koenker R, Bassett Jr G (1978) Regression quantiles. *Econometrica* **46(1)**:33–50. doi:10.2307/1913643.

Saerens M (2000) Building cost functions minimizing to some summary statistics. *IEEE Transactions on Neural Networks* **11(6)**:1263–1271. doi:10.1109/72.883416.

Thomson W (1979) Eliciting production possibilities from a well-informed manager. *Journal of Economic Theory* **20(3)**:360–380. doi:10.1016/00220531(79)900425.

Examples

```
# Compute the asymmetric piecewise linear scoring function (quantile scoring
# function).

df <- data.frame(
    y = rep(x = 0, times = 6),
    x = c(2, 2, -2, -2, 0, 0),
    p = rep(x = c(0.05, 0.95), times = 3)
)

df$quantile_penalty <- quantile_sf(x = df$x, y = df$y, p = df$p)

print(df)

# The absolute error scoring function is twice the asymmetric piecewise linear
# scoring function (quantile scoring function) at level p = 0.5.

df <- data.frame(
    y = rep(x = 0, times = 3),
    x = c(-2, 0, 2),
    p = rep(x = c(0.5), times = 3)
)

df$quantile_penalty <- quantile_sf(x = df$x, y = df$y, p = df$p)

df$absolute_error <- aerr_sf(x = df$x, y = df$y)

print(df)
```

---

relerr_sf — *Relative error scoring function (MAE-PROP scoring function)*

Description

The function relerr_sf computes the relative error scoring function when y materializes and x is the predictive $\text{med}^{(1)}(F)$ functional.

The relative error scoring function is defined in Table 1 in Gneiting (2011).

The relative error scoring function is referred to as MAE-PROP scoring function in eq. (13) in Patton (2011).

Usage

```
relerr_sf(x, y)
```

Arguments

x	Predictive $\text{med}^{(1)}(F)$ functional (prediction). It can be a vector of length n (must have the same length as y).

y Realization (true value) of process. It can be a vector of length n (must have the same length as x).

Details

The relative error scoring function is defined by:

$$S(x, y) := |(x - y)/x|$$

Domain of function:

$$x > 0$$

$$y > 0$$

Range of function:

$$S(x, y) \geq 0, \forall x, y > 0$$

Value

Vector of relative errors.

Note

For details on the relative error scoring function, see Gneiting (2011).

The β-median functional, $\text{med}^{(\beta)}(F)$ is the median of a probability distribution whose density is proportional to $y^\beta f(y)$, where f is the density of the probability distribution F of y (Gneiting 2011).

The relative error scoring function is negatively oriented (i.e. the smaller, the better).

The relative error scoring function is strictly consistent for the $\text{med}^{(1)}(F)$ functional relative to the family $\mathbb{F}$ of potential probability distributions (whose densities are proportional to $y f(y)$, where f is the density of the probability distribution F for the future y) (see Theorems 5 and 9 in Gneiting 2011).

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Patton AJ (2011) Volatility forecast comparison using imperfect volatility proxies. *Journal of Econometrics* **160(1)**:246–256. doi:10.1016/j.jeconom.2010.03.034.

Examples

```
# Compute the relative error scoring function.

df <- data.frame(
    y = rep(x = 2, times = 3),
    x = 1:3
)

df$relative_error <- relerr_sf(x = df$x, y = df$y)

print(df)
```

`serr_sf` *Squared error scoring function*

Description

The function serr_sf computes the squared error scoring function when y materializes and x is the predictive mean functional.

The squared error scoring function is defined in Table 1 in Gneiting (2011).

Usage

```
serr_sf(x, y)
```

Arguments

x	Predictive mean functional (prediction). It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).

Details

The squared error scoring function is defined by:

$$S(x, y) := (x - y)^2$$

Domain of function:

$$x \in \mathsf{R}$$

$$y \in \mathsf{R}$$

Range of function:

$$S(x, y) \geq 0, \forall x, y \in \mathsf{R}$$

Value

Vector of squared errors.

Note

For details on the squared error scoring function, see Savage 1971, Gneiting (2011).

The mean functional is the mean $E_F[Y]$ of the probability distribution F of y (Gneiting 2011).

The squared error scoring function is negatively oriented (i.e. the smaller, the better).

The squared error scoring function is strictly consistent for the mean functional relative to the family $\mathbb{F}$ of potential probability distributions F for the future y for which the second moment exists and is finite (Savage 1971, Gneiting 2011).

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Savage LJ (1971) Elicitation of personal probabilities and expectations. *Journal of the American Statistical Association* **66(337)**:783–810. doi:10.1080/01621459.1971.10482346.

Examples

```
# Compute the squarer error scoring function.

df <- data.frame(
    y = rep(x = 0, times = 5),
    x = -2:2
)

df$squared_error <- serr_sf(x = df$x, y = df$y)

print(df)
```

---

sperr_sf *Squared percentage error scoring function*

Description

The function sperr_sf computes the squared percentage error scoring function when y materializes and x is the predictive $\frac{\mathrm{E}_F[Y^{-1}]}{\mathrm{E}_F[Y^{-2}]}$ functional.

The squared percentage error scoring function is defined in p. 752 in Gneiting (2011).

Usage

```
sperr_sf(x, y)
```

Arguments

`x` Predictive $\frac{\mathrm{E}_F[Y^{-1}]}{\mathrm{E}_F[Y^{-2}]}$ functional (prediction). It can be a vector of length n (must have the same length as y).

`y` Realization (true value) of process. It can be a vector of length n (must have the same length as x).

Details

The squared percentage error scoring function is defined by:

$$S(x, y) := ((x - y)/y)^2$$

Domain of function:

$$x > 0$$

$$y > 0$$

Range of function:

$$S(x, y) \geq 0, \forall x, y > 0$$

Value

Vector of squared percentage errors.

Note

For details on the squared percentage error scoring function, see Park and Stefanski (1998) and Gneiting (2011).

The squared percentage error scoring function is negatively oriented (i.e. the smaller, the better).

The squared percentage error scoring function is strictly consistent for the $\frac{\mathrm{E}_F[Y^{-1}]}{\mathrm{E}_F[Y^{-2}]}$ functional.

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Park H, Stefanski LA (1998) Relative-error prediction. *Statistics and Probability Letters* **40(3)**:227–236. doi:10.1016/S01677152(98)000881.

Examples

```
# Compute the squared percentage error scoring function.

df <- data.frame(
    y = rep(x = 2, times = 3),
    x = 1:3
)

df$squared_percentage_error <- sperr_sf(x = df$x, y = df$y)

print(df)
```

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srelerr_sf *Squared relative error scoring function*

Description

The function srelerr_sf computes the squared relative error scoring function when y materializes and x is the predictive $\frac{\mathrm{E}_F[Y^2]}{\mathrm{E}_F[Y]}$ functional.

The squared relative error scoring function is defined in p. 752 in Gneiting (2011).

Usage

```
srelerr_sf(x, y)
```

Arguments

x	Predictive $\frac{\mathrm{E}_F[Y^2]}{\mathrm{E}_F[Y]}$ functional (prediction). It can be a vector of length n (must have the same length as y).
y	Realization (true value) of process. It can be a vector of length n (must have the same length as x).

Details

The squared relative error scoring function is defined by:

$$S(x, y) := ((x - y)/x)^2$$

Domain of function:

$$x > 0$$

$$y > 0$$

Range of function:

$$S(x, y) \geq 0, \forall x, y > 0$$

Value

Vector of squared relative errors.

Note

For details on the squared relative error scoring function, see Gneiting (2011).

The squared relative error scoring function is negatively oriented (i.e. the smaller, the better).

The squared relative error scoring function is strictly consistent for the $\frac{\mathrm{E}_F[Y^2]}{\mathrm{E}_F[Y]}$ functional.

References

Gneiting T (2011) Making and evaluating point forecasts. *Journal of the American Statistical Association* **106(494)**:746–762. doi:10.1198/jasa.2011.r10138.

Examples

```
# Compute the squared percentage error scoring function.

df <- data.frame(
    y = rep(x = 2, times = 3),
    x = 1:3
)

df$squared_relative_error <- srelerr_sf(x = df$x, y = df$y)

print(df)
```

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